Delay-Range-Dependent Guaranteed Cost Control for Batch Processes with State Delay

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A guaranteed cost control scheme is proposed for batch processes described by a two-dimensional (2-D) system with uncertainties and interval time-varying delay. First, a 2-D controller, which includes a robust feedback control to ensure performances over time and an iterative learning control to improve the tracking performance from cycle to cycle, is formulated. The guaranteed cost law concept of the proposed 2-D controller is then introduced. Subsequently, by introducing the Lyapunov–Krasovskii function and adding a differential inequality to the Lyapunov function for the 2-D system, sufficient conditions for the existence of the robust guaranteed cost controller are derived in terms of matrix inequalities. A design procedure for the controller is also presented. Furthermore, a convex optimization problem with linear matrix inequality (LMI) constraints is formulated to design the optimal guaranteed cost controller that minimizes the upper bound of the closed-loop system cost. The proposed control law can stabilize the closed-loop system as well as guarantee H_∞ *performance level and a cost function with upper bounds for all admissible uncertainties. The results can be easily extended to the constant delay case. Finally, an illustrative example is given to demonstrate the effectiveness and advantages of the proposed 2-D design approach.* *AIChE J*, 59: 2033–2045, 2013
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Introduction

Advanced control is critically important in ensuring the quality and quality consistency of batch processes, because a batch process is the preferred choice for manufacturing high-value-added products. Compared with a continuous process, a batch process has unique characteristics, one of which is its repetitive nature. Iterative learning control (ILC) has been used widely to exploit the repetitive nature of batch processes. A conventional ILC scheme works as an open-loop feed-forward compensator. An ILC that does not use within-cycle feedback has been found to have the tendency to be sensitive to perturbation and slow in system convergence.[1]

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To mitigate the abovementioned problem, a feedback controller for use with an ILC scheme is proposed. The result is the so-called feedback feed-forward ILC, that is, 2-D control that determines the dynamic behavior along the time and cycle directions.[2] The 2-D controller design problem can be transformed into a stabilization scheme for a special 2-D system. Some 2-D controller design results have already been developed based on the 2-D system theory, including ILC design for batch processes with uncertainties,[2–4] iterative learning fault-tolerant (reliable) control for batch processes with actuator or sensor faults,[5,6] iterative learning model predictive control for batch processes,[7,8] and so on. However, these outputs were limited to the system model without time-delay.

Time-delay, which is a phenomenon that exists widely in chemical and other industrial processes, is a source of instability and performance degradation in numerous control systems. For continuous processes, the control problem for time-delay systems has been a hot topic in the control community and has received a great deal of attention.[9–11] The effect of time delay can increase the complexity of stable system analysis because of the characteristics of batch processes. Studying the stability of batch processes is very important. However, insufficient attention has been directed toward this topic to date, and results on the subjects remain limited.

In the frequency domain, an ILC algorithm[12] was proposed using the Smith predictor control structure to improve the tracking performance of ILC for batch processes with obvious time delay. An ILC scheme based on the internal model control structure was proposed for batch processes with model uncertainties, including time delay mismatch.[13] A reference shift algorithm based on a double-loop ILC structure[14] was suggested to address the output delay of a nonminimum phase plant. In the time domain, an ILC method for a holding mechanism was proposed for the control input during the estimated time delay for the operation of batch processes with time delay.[15] In these existing results, only fixed time-delay was considered with a conventional ILC method.

A batch process often suffers from time-varying delay, which may result in the instability of a control system or in the degradation of system performance. Determining a control design method for batch processes with time-varying delay remains a challenge. Feedback control incorporated with the ILC technique has been demonstrated as an effective tool for improving the stability of batch processes without time delay.

To the best knowledge of the authors, few control results on batch processes with time-varying delay within a range have been made available. The authors[16,17] propose a robust feedback integrated with ILC scheme for time-varying batch processes using a 2-D Rosser model based on the general 2-D system theory.[18,19] Based on a 2-D system description of a batch process, a robust closed-loop ILC associated with an output feedback scheme is proposed for batch processes with state delay and time-varying uncertainties.[20]

Two major issues revolve around the robust controller design. The first is concerned with the robust stability of the uncertain closed-loop system, whereas the other is geared toward robust performance. Note that the latter is more important, because designing a control system that is stable and has a guaranteed adequate level of performance is always desirable when controlling a system that is dependent on uncertain parameters. Given the so-called guaranteed cost control approach first introduced by Chang and Peng,[21] this issue on one-dimensional (1-D) systems has been addressed extensively, extending to the study of time-delay systems over the past years. Examples include Zhao et al.[22] for continuous time case and Yu and Gao[23] for discrete time case.

In recent years, 2-D discrete systems have been applied in many areas. The guaranteed cost control problem for 2-D discrete uncertain systems has received considerable attention. Meanwhile, a robust controller design method has been established. Furthermore, an optimization problem is introduced to select the suboptimal guaranteed cost controller that minimizes the upper bound of the cost function,[24–26] and research results have been extended to include time-delay systems.[27,28] However, the general 2-D discrete system with interval time-varying delay is yet to be addressed in the literature. The results about using 2-D system theory to study the batch process are very limited, too.

This article primarily deals with the guaranteed cost control problem for batch processes with uncertainties and state delay varying in a range. We first propose a 2-D controller and then establish a model similar to a 2-D Fornasini–Marchsini model with a delay that varies in a range serving as a process model for the proposed design. LMI-based delay-range-dependent sufficient conditions for the existence of guaranteed cost 2-D control laws, and the feasible solutions to this LMI are used to construct the guaranteed cost controllers. Furthermore, a convex optimization problem is introduced to determine the optimal guaranteed cost controller that minimizes the upper bound on the cost function. To solve the nonrepeatable perturbation, the guaranteed cost 2-D control laws will satisfy the H_∞ performance. In addition, the results are extended to the constant time delay case, which makes the study more general. There main contributions of this work can be summarized as follows: (1) An ILC time-delay system is represented equivalently as a 2-D model with a delay varying in a range. Based on this, the robust convergences that are related to the ILC system can be translated to robust stabilities that are related to a 2-D system. This provides a foundation for the design and analysis of an ILC system based on a 2-D system. (2) According to bounded parameter perturbations, a 2-D-ILC guaranteed cost controller, which satisfies H_∞ performance level and a cost function with upper bounds for all admissible uncertainties, is first constructed. (3) Different from the existing results that can be found in the pertinent literature, delay-range-dependent sufficient conditions are derived without introducing redundant matrix variables, which will result in a smaller amount of calculation and fewer constraints. Meanwhile, the conditions extend to constant delay case. This makes the conditions more universal significance and broader applications. Simulation shows that the proposed 2-D controller design method effectively meets the design objectives.

Throughout this article, the following notations are used: R^n represents Euclidean n space, with the norm denoted by $\|\bullet\|$. For any matrix M, $M > 0$ ($M < 0$) indicates that M is a positive (negative) definite matrix. M^T represents the transposed matrix M. I and $\mathbf{0}$, respectively, denote the identity and zero matrices with appropriate dimensions. The asterisk notation (*) represents the symmetric element of a matrix. $|\bullet|$ denotes the absolute value of "•." For a 2-D signal $\omega(t,k)$, if $\|\omega(\cdot,\cdot)\|_{2D-2e} = \sqrt{\sum_{t=0}^{N_1}\sum_{k=0}^{N_2}\|\omega(\cdot,\cdot)\|^2} < \infty$ for any integers $N_1, N_2 > 0$, then $\omega(t,k)$ is said to be in ℓ_{2D-2e} space, as denoted by $\omega(\cdot,\cdot) \in \ell_{2D-2e}$.

Problem Description and 2-D System Representation

Problem description

Consider process $\Sigma_{\text{P-delay}}$ with the repetitive performance of a task over a certain period (called a cycle) described by the following discrete-time model with uncertain parameter perturbations and interval time-varying delay

$$\sum_{\text{P-delay}}:\begin{cases} x(t+1,k+1)=A(t)x(t,k+1)+A_d(t)x(t-d(t),k+1)+B(t)u(t,k+1) \\ y(t,k+1)=Cx(t,k+1) \\ x(0,k+1)=x_{0,k+1}, \quad 0\le t\le T,\ k=0,1,2,\ldots \end{cases} \tag{1}$$

where t denotes time, k denotes the cycle index, and $x_{0,k+1}$ is the initial condition of the $(k+1)$th batch run. $x(t,k+1)\in R^n$, $y(t,k+1)\in R^l$, and $u(t,k+1)\in R^m$ represent the state, output, and input of the process, respectively, at time t in the $(k+1)$th batch run. The time-varying delay $d(t)$ along the horizontal direction satisfies

$$d_m \le d(t) \le d_M \tag{2}$$

where d_m and d_M denote the lower and upper delay bounds, respectively. $A(t)=A+\Delta_a(t)$, $A_d(t)=A_d+\Delta_d(t)$, $B(t)=B+\Delta_b(t)$, and $\{A,A_d,B,C\}$ are constant matrices of appropriate dimensions, and $\{\Delta_a(t),\Delta_d(t),\Delta_b(t)\}$ are some perturbations and may be specified as $[\Delta_a(t)\Delta_d(t)\Delta_b(t)]=E\Delta(t,k+1)$ $[F\ F_d\ F_b]$ with $\Delta^T(t,k+1)\Delta(t,k+1)\le I,\ 0\le t\le T$; $k=1,2,\ldots$, where $\{E,F_d,F_b\}$ are known constant matrices. Note that $\Delta(t,\ k+1)$ is generally represented as the functions of both time t and cycle $(k+1)$. If $\Delta(t,\ k+1)$ depends on time t only, then the uncertain parameter perturbations are called repeatable. Otherwise, the uncertain parameter perturbations are referred to as nonrepeatable. Although conventional ILC schemes can effectively address repeatable parameter perturbations, considering the control of batch processes with nonrepeatable parameter perturbations is equally important.

Equivalent 2-D system representation

For the uncertain batch process $\Sigma_{\text{P-delay}}$ that is described by (1), an ILC law is introduced with the following form

$$\sum_{\text{ILC}}: u(t,k+1)=u(t,k)+r(t,k+1) \quad (\text{for } u(t,0)=0, t=0,1,2,\cdots,T) \tag{3}$$

where $u(t,0)$ is the initial profile of iteration, and $r(t,k+1)\in R^m$ is called the updating law of the ILC to be determined. The objective of this article is to determine the updating law $r(t,\ k+1)$, such that $y(t,\ k+1)$ tracks the setpoint trajectory $y_d(t)$ perfectly for each cycle. Define the output tracking error in the current cycle as

$$e(t,k+1)=y(t,k+1)-y_d(t) \tag{4}$$

Meanwhile, define a batchwise direction function of error as

$$\delta(f(t,k+1))=f(t,k+1)-f(t,k) \tag{5}$$

where $\delta(f(t,\ k+1))$ is state variable or output variable, and, thus, is in accordance with the ILC updating law as shown in (3).

From Models (1), (3) and (4)–(5), the following can be derived

$$\begin{cases} \delta(x(t+1,k+1))=A(t)\delta(x(t,k+1))+A_d(t)\delta(x(t-d(t),k+1))+B(t)r(t,k+1)+\omega(t,k+1) \\ e(t+1,k+1)=e(t+1,k)+\delta(x(t+1,k+1)) \\ =e(t+1,k)+CA(t)\delta(x(t,k+1))+CA_d(t)\delta(x(t-d(t),k+1))+CB(t)r(t,k+1)+C\omega(t,k+1) \end{cases} \tag{6}$$

where

$$\omega(t,k+1)=(\Delta_a(t,k+1)-\Delta_a(t,k))x(t,k)+(\Delta_d(t,k+1)-\Delta_d(t,k))x(t-d(t),k)+(\Delta_b(t,k+1)-\Delta_b(t,k))u(t,k)$$

The augmented 2-D model can then be obtained as

$$\sum_{\text{2D-P-delay}}:\begin{cases} x_1(t+1,k+1)=\tilde{A}_1x_1(t,k+1)+\tilde{A}_2x_1(t+1,k) \\ +\tilde{A}_dx_1(t-d(t),k+1)+\tilde{B}r(t,k+1)+\tilde{H}\omega(t,k+1) \\ z(t,k+1)\triangleq e(t,k+1)=\tilde{G}x_1(t,k+1) \end{cases} \tag{7}$$

where $x_1(t,k)=\begin{bmatrix}\delta(x(t,k))\\ e(t,k)\end{bmatrix}$, $\tilde{A}_1=\begin{bmatrix}A(t) & \mathbf{0}\\ CA(t) & \mathbf{0}\end{bmatrix}=\bar{A}_1+\bar{\Delta}_a(t)$, $\tilde{A}_2=\bar{A}_2=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & I\end{bmatrix}$, $\tilde{A}_d=\begin{bmatrix}A_d(t) & \mathbf{0}\\ CA_d(t) & \mathbf{0}\end{bmatrix}=\bar{A}_d+\bar{\Delta}_d(t)$, $\tilde{B}=\begin{bmatrix}B(t)\\ CB(t)\end{bmatrix}=\bar{B}+\bar{\Delta}_b(t)$, $\bar{A}_1=\begin{bmatrix}A & \mathbf{0}\\ CA & \mathbf{0}\end{bmatrix}$, $\bar{A}_d=\begin{bmatrix}A_d & \mathbf{0}\\ CA_d & \mathbf{0}\end{bmatrix}$, $\bar{B}=\begin{bmatrix}B\\ CB\end{bmatrix}$, $\bar{\Delta}_a(t)=\bar{E}\Delta(t,k)\bar{F}$, $\bar{\Delta}_d(t)=\bar{E}\Delta(t,k)\bar{F}_d$, $\bar{\Delta}_b(t)=\bar{E}\Delta(t,k)F_b$ $\bar{E}=\begin{bmatrix}E\\ CE\end{bmatrix}$, $\bar{F}=[F\quad \mathbf{0}]$, $\bar{F}_d=[F_d\quad \mathbf{0}]$, $\tilde{H}=\begin{bmatrix}I\\ C\end{bmatrix}$, and $\tilde{G}=[\mathbf{0}\quad I]$.

Model $\Sigma_{\text{2D-P-delay}}$ equivalently represents the dynamic behavior of the tracking error of the system that is described by (1). Hence, this model is called the equivalent 2-D tracking error model of system (1). The design of the updating law $r(t,\ k+1)$ for system (1) is clearly equivalent to the design of a 2-D control law for the equivalent 2-D tracking error model $\Sigma_{\text{2D-P-delay}}$. Here, model $\Sigma_{\text{2D-P-delay}}$ (7) is a special 2-D system that is distinct from the general uncertain 2-D time-delay systems in that the information propagation in the time direction only occurs over a finite duration. Designing a controller such that the new special 2-D system is controllable and preserves an adequate control performance is important.

For 2-D system $\Sigma_{2D\text{-}P\text{-}delay}$, the boundary conditions are of two dimensions and are assumed to satisfy the following form

$$\begin{cases} x_1(t,0)=\mu_{t0}, & \forall 0 \le t < r_1, \\ x_1(0,k)=\upsilon_{0k}, & \forall 0 \le k < r_2, \\ \mu_{00}=\upsilon_{00} \end{cases} \tag{8}$$

where $r_1 < \infty$ and $r_2 < \infty$ are positive integers, and μ_{t0} and υ_{0k} are given vectors. Here, $\mu_{0k} = \upsilon_{t0} = 0$ is called the zero boundary condition. To design the robust 2-D guaranteed cost control law and analyze the robust guaranteed cost performance of the closed-loop system, a number of definitions and lemmas are first introduced.

Definition 1. For any bounded boundary conditions (8), define $\chi_l = \sup\{\|x_1(t,k)\| : t+k=l, t,k \ge 1\}$; if $\lim_{l\to\infty}\chi_l=0$, the 2-D system $\Sigma_{2D\text{-}P\text{-}delay}$ (7) with $r(t, k+1) = 0$ and $\omega(t, k+1) = 0$ is said to be asymptotically stable.

Associated with 2-D system (7) is the following cost function

$$J=\sum_{t=0}^{N_1}\sum_{k=0}^{N_2}[x_1^T(t,k+1)Q_1x_1(t,k+1)+x_1^T(t+1,k)Q_2x_1(t+1,k) + r^T(t,k+1)Ur(t,k+1)] \tag{9}$$

where $Q_1 > 0$, $Q_2 > 0$, and $U > 0$.

Remark 1. In the above performance index function, the relative size of the weighting matrices Q_1, Q_2, and U directly affects the performance of the closed-loop control system. For example, the optimization scheme determined by relatively small U allows the control signal along the periodic direction having relatively large variations, which help to improve the speed of convergence of ILC system along the periodic direction, but may reduce the robust stability of the control system. Meanwhile, the size of matrix values Q_1, Q_2, and U will directly affect the following theorems whether they are solvable or not. Therefore, matrices Q_1, Q_2, and U must be positive and be adjusted to the appropriate size.

Definition 2. Consider the uncertain 2-D system (7) with $\omega(t, k+1) = 0$ and the cost function (9), if a controller $r^*(t, k+1)$ and a positive scalar J^* exist, such that for all admissible uncertainties, the resulting closed-loop system is stable and its cost function (9) satisfies $J \le J^*$, then J^* is said to be a guaranteed cost, and $r^*(t, k+1)$ is said to be a guaranteed cost controller for the uncertain 2-D system (7).

Definition 3. For a given scalar $\gamma > 0$, control law $r^*(t, k+1)$ is a robust H_∞ guaranteed cost law for the uncertain 2-D system (7), if the following conditions hold for all admissible parameter uncertainties:

1. the resulting closed-loop system (7) with $\omega(t, k+1) = 0$ is asymptotically stable;
2. with the zero initial condition, the controlled output $z(t, k+1)$ satisfies $\|z\|_{2D-2e} \le \gamma\|\omega\|_{2D-2e}$;
3. the cost function for the resulting closed-loop system (7) satisfies $J \le J^*$ when $\omega(t, k+1) = 0$.

Remark 2. For nonrepetitive perturbations, it can be seen from system (7) that we only need to let internal disturbances of the system (1) converted to external disturbances, and thereby take into account H_∞ control problem. The robust H_∞ performance represents the upper bound of the sensitivity of the controlled output to the disturbance. Smaller values of γ indicate smaller sensitivity or better rejection performance to the disturbance. It is inevitable to utilize robust H_∞ performance to analyze the impacts of the cycle-to-cycle parameter perturbations to the control performances of the system (7).

Lemma 1. [18] For any vector $\delta(t) \in R^n$, two positive integers κ_0 and κ_1, and matrix $0 < R \in R^{n\times n}$, the following inequality holds

$$-(\kappa_1-\kappa_0+1)\sum_{t=\kappa_0}^{\kappa_1}\delta^T(t)R\delta(t) \le -\sum_{t=\kappa_0}^{\kappa_1}\delta^T(t)R\sum_{t=\kappa_0}^{\kappa_1}\delta(t) \tag{10}$$

The objective of this article is to design a 2-D robust guaranteed cost controller

$$\sum_{2D\text{-}C\text{-}delay} : r(t,k+1)=K_1x_1(t,k+1)+K_2x_1(t+1,k) \tag{11}$$

that can be used to stabilize system (7) and achieve a small value of J^* for any delay satisfying (2). Applying the controller (11) to system (7) will result in the closed-loop system

$$\sum_{2D\text{-}P\text{-}delay\text{-}C}:$$

$$\begin{cases} x_1(t+1,k+1)=\tilde{A}_{1k}x_1(t,k+1)+\tilde{A}_{2k}x_1(t+1,k) \\ +\tilde{A}_d x_1(t-d(t),k+1)+\tilde{H}\omega(t,k+1)z(t,k+1) \\ \triangleq e(t,k+1)=\tilde{G}x_1(t,k+1) \end{cases} \tag{12}$$

where $\tilde{A}_{1k}=\tilde{A}_1+\tilde{B}K_1$ and $\tilde{A}_{2k}=\tilde{A}_2+\tilde{B}K_2$.

Main Results

Robust guaranteed cost performance analysis

In this section, sufficient conditions for the existence of the robust guaranteed cost control law for 2-D systems based on repetitive perturbation and nonrepetitive perturbation, respectively, are conducted. The results can be represented by the following theorem:

Theorem 1 Consider the 2-D system (7) with the initial conditions (8); for some given scalars $0 \le d_m \le d_M$, if symmetric positive matrices P, Q, W, and $R \in R^{(n+l)\times(n+l)}$ and a positive scale $\alpha < 1$ exist, such that the following matrix inequality as shown in (13b) holds, then for any delay $d(t)$ satisfying $0 \le d_m \le d(t) \le d_M$, control law $r(t,k+1)=K_1x_1(t,k+1)+K_2x_1(t+1,k)$ is a guaranteed cost controller, and the cost function (9) of the resulting closed-loop 2-D system $\Sigma_{2D\text{-}P\text{-}delay\text{-}C}$ (12) with $\omega(t, k+1) = 0$ satisfies the following upper bound

$$J \le \sum_{k=0}^{N_2}[x_1^T(0,k+1)\alpha Px_1(0,k+1)+\sum_{r=-d(0)}^{-1}x_1^T(r,k+1)Qx_1(r,k+1)+\sum_{r=-d_M}^{-1}x_1^T(r,k+1)Wx_1(r,k+1) + \sum_{s=-d_M}^{-d_m}\sum_{r=s}^{-1}x_1^T(r,k+1)Qx_1(r,k+1)+d_M\sum_{s=-d_M}^{-1}\sum_{r=s}^{-1}\eta^T(r,k+1)R\eta(r,k+1)]+\sum_{t=0}^{N_1}x_1^T(t+1,0)(1-\alpha)Px_1(t+1,0) \tag{13a}$$

$$\begin{bmatrix} \Psi_1 & \Psi_2 \\ \Psi_2^T & \Psi_3 \end{bmatrix} < 0 \tag{13b}$$

where
$$\Psi_1 = \begin{bmatrix} \Psi_{11} & \mathbf{0} & \mathbf{0} & R \\ \mathbf{0} & -(1-\alpha)P+Q_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -Q & \mathbf{0} \\ R & \mathbf{0} & \mathbf{0} & -W-R \end{bmatrix}$$ with
$\Psi_{11} = -\alpha P + W + (d_M - d_m + 1)Q + Q_1 - R$, and
$$\Psi_2 = \begin{bmatrix} K_1^T & \tilde{A}_{1k}^T P & (\tilde{A}_{1k}-I)^T R \\ K_2^T & \tilde{A}_{2k}^T P & \tilde{A}_{2k}^T R \\ \mathbf{0} & \tilde{A}_d^T P & \tilde{A}_d^T R \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$
$\Psi_3 = -\mathrm{diag}[\,U^{-1} \quad P \quad d_M^{-2} R].$

Proof:. Select the following Lyapunov function candidate

$$V(t+\theta,k+\tau) = V_h(t+\theta,k+\tau) + V_v(t+\theta,k+\tau) \tag{14a}$$

where
$V_h(t+\theta,k+\tau) = \Sigma_{l=1}^{5} V_l(t+\theta,k+\tau)$,
$V_1(t+\theta,k+\tau) = x_1^T(t+\theta,k+\tau)\alpha P x_1(t+\theta,k+\tau)$,
$V_2(t+\theta,k+\tau) = \Sigma_{r=t+\theta-d(t+\theta)}^{t+\theta-1} x_1^T(r,k+\tau)Qx_1(r,k+\tau)$,
$V_3(t+\theta,k+\tau) = \Sigma_{r=t+\theta-d_M}^{t+\theta-1} x_1^T(r,k+\tau)Wx_1(r,k+\tau)$,
$V_4(t+\theta,k+\tau) = \Sigma_{s=-d_M}^{-d_m} \Sigma_{r=t+\theta+s}^{t+\theta-1} x_1^T(r,k+\tau)Qx_1(r,k+\tau)$,
$V_5(t+\theta,k+\tau) = d_M \Sigma_{s=-d_M}^{-1} \Sigma_{r=t+\theta+s}^{t+\theta-1} \eta^T(r,k+\tau)R\eta(r,k+\tau)$,
$V_v(t+\theta,k+\tau) = x_1^T(t+\theta,k+\tau)(1-\alpha)Px_1(t+\theta,k+\tau)$, and $\eta(r,k+1) = x_1(r+1,k+1) - x_1(r,k+1)$, and P, Q, W, and R are positive definite matrices to be determined. Designing

$$\Delta V(t+1,k+1) = V_h(t+1,k+1) - V_h(t,k+1) + V_v(t+1,k+1) - V_v(t+1,k) = \sum_{l=1}^{5} \Delta V_l(t+1,k+1) + \Delta V_v(t+1,k+1) \tag{14b}$$

yields

$$\Delta V_1(t+1,k+1) = x_1^T(t+1,k+1)\alpha P x_1(t+1,k+1) - x_1^T(t,k+1)\alpha P x_1(t,k+1) \tag{15a}$$

$$\Delta V_2(t+1,k+1) \le x_1^T(t,k+1)Qx_1(t,k+1) - x_1^T(t-d(t),k+1) Qx_1(t-d(t),k+1) + \sum_{r=t-d_M}^{t-d_m} x_1^T(r,k+1)Qx_1(r,k+1) \tag{15b}$$

$$\Delta V_3(t+1,k+1) = x_1^T(t,k+1)Wx_1(t,k+1) - x_1^T(t-d_M,k+1)Wx_1(t-d_M,k+1) \tag{15c}$$

$$\Delta V_4(t+1,k+1) = (d_M - d_m)x_1^T(t,k+1)Qx_1(t,k+1) - \sum_{r=t-d_M}^{t-d_m} x_1^T(r,k+1)Qx_1(r,k+1) \tag{15d}$$

$$\Delta V_5(t+1,k+1) = d_M^2 \eta^T(t,k+1)R\eta(t,k+1) - d_M \sum_{r=t-d_M}^{t-1} \eta^T(r,k+1)R\eta(r,k+1) \tag{15e}$$

and

$$\Delta V_v(t+1,k+1) = x_1^T(t+1,k+1)(1-\alpha)Px_1(t+1,k+1) - x_1^T(t+1,k)(1-\alpha)Px_1(t+1,k) \tag{15f}$$

It follows from Lemma 1 that

$$\begin{aligned} \Delta V_5(t+1,k+1) \le{}& d_M^2 \eta^T(t,k+1)R\eta(t,k+1) - \sum_{r=t-d_M}^{t-1} \eta^T(r,k+1) R \sum_{r=t-d_M}^{t-1} \eta(r,k+1) \\ ={}& d_M^2 (x_1(t+1,k+1) - x_1(t,k+1))^T R (x_1(t+1,k+1) - x_1(t,k+1)) \\ &- (x_1(t,k+1) - x_1(t-d_M,k+1))^T R (x_1(t,k+1) - x_1(t-d_M,k+1)) \end{aligned} \tag{16}$$

From (14b) and (15), we obtain

$$\Delta V(t+1,k+1) + \varphi_1^T(t,k) \begin{bmatrix} Q_1 + K_1^T U K_1 & K_1^T U K_2 \\ K_2^T U K_1 & Q_2 + K_2^T U K_2 \end{bmatrix} \varphi_1(t,k) \le \varphi^T(t,k) \left(\begin{bmatrix} \Psi_{11} + K_1^T U K_1 & K_1^T U K_2 & \mathbf{0} & R \\ K_2^T U K_1 & -(1-\alpha)P + Q_2 + K_2^T U K_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -Q & \mathbf{0} \\ R & \mathbf{0} & \mathbf{0} & -W-R \end{bmatrix} + \begin{bmatrix} \tilde{A}_{1k}^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \end{bmatrix} P \begin{bmatrix} \tilde{A}_{1k}^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \end{bmatrix}^T + \begin{bmatrix} (\tilde{A}_{1k}-I)^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \end{bmatrix} d_M^2 R \begin{bmatrix} (\tilde{A}_{1k}-I)^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \end{bmatrix}^T \right) \varphi(t,k) \tag{17a}$$

where $\varphi^T(t,k) = [\,\varphi_1^T(t,k) \quad \varphi_2^T(t,k)\,]$, $\varphi_1^T(t,k) = [\,x_1^T(t,k+1) \; x_1^T(t+1,k)]$, and $\varphi_2^T(t,k) = [\,x_1^T(t-d(t),k+1) \;\; x_1^T(t-d_M,k+1)]$.

Applying the Schur complement, the matrix inequality (13b) yields

$$\Delta V(t+1,k+1) \le -\varphi_1^T(t,k) \begin{bmatrix} Q_1 + K_1^T U K_1 & K_1^T U K_2 \\ K_2^T U K_1 & Q_2 + K_2^T U K_2 \end{bmatrix} \varphi_1(t,k) \tag{17b}$$

which holds for any delay $d(t)$ satisfying $0 \le d_m \le d(t) \le d_M$. Given that $T\begin{bmatrix} Q_1 + K_1^T U K_1 & K_1^T U K_2 \\ K_2^T U K_1 & Q_2 + K_2^T U K_2 \end{bmatrix} > 0$, the following inequality is effective

$$V_h(t+1,k+1) + V_v(t+1,k+1) \le V_h(t,k+1) + V_v(t+1,k) \tag{18}$$

where the equality sign only holds when $\varphi(t,\, k) = 0$.

Define the set $D(r)$by $D(r) \triangleq \{(t,k) : t+k = r, t \ge 0, k \ge 0\}$. For any integer $r \ge \max\{r_1, r_2\}$, Eq. (15a) associated with the boundary condition (8) yields

$$\begin{aligned} \sum_{t+k \in D(r)} V(t,k) ={}& \sum_{t+k \in D(r)} [V_h(t,k) + V_v(t,k)] \\ ={}& V_h(r,0) + V_h(r-1,1) + \cdots + V_h(1,r-1) + V_h(0,r) \\ &+ V_v(r,0) + V_v(r-1,1) + \cdots + V_v(1,r-1) + V_v(0,r) \\ \ge{}& V_h(r+1,0) + V_h(r,1) + \cdots + V_h(1,r) + V_h(0,r+1) \\ &+ V_v(r+1,0) + V_v(r,1) + \cdots + V_v(1,r) + V_v(0,r+1) \\ \ge{}& \sum_{t+k \in D(r+1)} V(t,k) \end{aligned} \tag{19a}$$

where the equality sign only holds when $\sum_{t+k \in D(r)} V(t,k) = 0$. The sum of the Lyapunov function value clearly decreases along the state trajectories. Thus, we obtain

$$\lim_{t+k\to\infty} x_1(t,k) \to 0 \tag{19b}$$

Consequently, we conclude from Definition 1 that the closed-loop system (12) is asymptotically stable for any delay $d(t)$ satisfying $0 \le d_m \le d(t) \le d_M$.

Given that the inequality (17b) holds, the following can be obtained

$$\sum_{t=0}^{N_1}\sum_{k=0}^{N_2} \Delta V(t+1,k+1) \le \sum_{t=0}^{N_1}\sum_{k=0}^{N_2} -\varphi_1^T(t,k)\begin{bmatrix} Q_1+K_1^TUK_1 & K_1^TUK_2 \\ K_2^TUK_1 & Q_2+K_2^TUK_2 \end{bmatrix}\varphi_1(t,k) \tag{20a}$$

It follows from (20a) that

$$\sum_{k=0}^{N_2}(V_h(N_1,k)-V_h(0,k+1))+\sum_{t=0}^{N_1}(V_v(t+1,N_2)-V_v(t+1,0)) \le \sum_{t=0}^{N_1}\sum_{k=0}^{N_2} -\varphi_1^T(t,k)\begin{bmatrix} Q_1+K_1^TUK_1 & K_1^TUK_2 \\ K_2^TUK_1 & Q_2+K_2^TUK_2 \end{bmatrix}\varphi_1(t,k) \tag{20b}$$

From Definition 1 and the boundary conditions, when $N_1, N_2 \to \infty$, we obtain

$$\sum_{t=0}^{N_1}\sum_{k=0}^{N_2} \varphi_1^T(t,k)\begin{bmatrix} Q_1+K_1^TUK_1 & K_1^TUK_2 \\ K_2^TUK_1 & Q_2+K_2^TUK_2 \end{bmatrix}\varphi_1(t,k) \le \sum_{k=0}^{N_2} V_h(0,k+1)+\sum_{t=0}^{N_1} V_v(t+1,0) \tag{20c}$$

Therefore, based on Definition 2, the result of this theorem is true. ■

Remark 3. Different from the existing literature, a new Lyapunov–Krasovskii functional, which makes full use of the information of both the lower and the upper bounds of the interval time-varying delay, is proposed. The authors avoid introducing some redundant free weighting matrices that apart from the Lyapunov–Krasovskii functional matrices. Thereby, some novel delay-range-dependent stability criteria, which include a smaller amount of calculation and fewer constraints, are derived. Meanwhile, the functional may degenerate into constant delays; the delay considered in this article is systematical and comprehensive.

Theorem 2 For a prescribed constant $\gamma > 0$ and some given scalars $0 \le d_m \le d_M$, the 2-D control law $r(t,k+1)=K_1x_1(t,k+1)+K_2x_1(t+1,k)$ is a robust H_∞ guaranteed cost control law for system (7) if symmetric positive matrices P, Q, W, and $R \in R^{(n+l)\times(n+l)}$ and a positive scale $\alpha < 1$ exist such that the matrix inequality shown in (21) holds. The cost function then satisfies the upper bound (13a).

$$\begin{bmatrix} \hat{\Psi}_1 & \hat{\Psi}_2 & \hat{G}^T \\ \hat{\Psi}_2^T & \Psi_3 & \mathbf{0} \\ \hat{G} & \mathbf{0} & -I \end{bmatrix} < 0 \tag{21}$$

where $\hat{\Psi}_1=\begin{bmatrix} \Psi_1 & \mathbf{0} \\ \mathbf{0} & -\gamma^2 I \end{bmatrix}$, $\hat{\Psi}_2=\begin{bmatrix} \Psi_2 \\ \hat{H} \end{bmatrix}$, $\hat{G}=\begin{bmatrix} \tilde{G} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$, and $\hat{H}=\begin{bmatrix} \mathbf{0} & \tilde{H}^TP & \tilde{H}^TP \end{bmatrix}$.

Proof: To establish the H_∞ guaranteed cost performance analysis of the 2-D system (9) with zero boundary conditions for any nonzero $\omega(t,k) \in l_2\{[0,\infty],[0,\infty]\}$, we define

$$J_1=\sum_{t=0}^{N_1}\sum_{k=0}^{N_2}[z^T(t,k+1)z(t,k+1)-\gamma^2\omega^T(t,k+1)\omega(t,k+1)] \tag{22a}$$

Then, for any nonzero $\omega(t,k) \in l_2\{[0,\infty],[0,\infty]\}$

$$J_1 \le \sum_{t=0}^{N_1}\sum_{k=0}^{N_2}[z^T(t,k+1)z(t,k+1)-\gamma^2\omega^T(t,k+1)\omega(t,k+1) +\Delta V(t+1,k)] \tag{22b}$$

However

$$z^T(t,k+1)z(t,k+1)-\gamma^2\omega^T(t,k+1)\omega(t,k+1)+\Delta V(t+1,k) = \begin{bmatrix} \varphi(t,k) \\ \omega(t,k+1) \end{bmatrix}^T \Gamma \begin{bmatrix} \varphi(t,k) \\ \omega(t,k+1) \end{bmatrix} \tag{22c}$$

where

$$\Gamma=\begin{bmatrix} -\alpha P+W+(d_M-d_m+1)Q-R+\tilde{G}^T\tilde{G} & \mathbf{0} & \mathbf{0} & R & \mathbf{0} \\ \mathbf{0} & -(1-\alpha)P & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -Q & \mathbf{0} & \mathbf{0} \\ R & \mathbf{0} & \mathbf{0} & -W-R & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \tilde{A}_{1k}^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \\ \tilde{H}^T \end{bmatrix} P \begin{bmatrix} \tilde{A}_{1k}^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \\ \tilde{H}^T \end{bmatrix}^T + \begin{bmatrix} (\tilde{A}_{1k}-I)^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \\ \tilde{H}^T \end{bmatrix} d_M^2 R \begin{bmatrix} (\tilde{A}_{1k}-I)^T \\ \tilde{A}_{2k}^T \\ \tilde{A}_d^T \\ \mathbf{0} \\ \tilde{H}^T \end{bmatrix}^T.$$

Therefore

$$J_1 \le \sum_{t=0}^{N_1}\sum_{k=0}^{N_2}\left[\begin{bmatrix} \varphi(t,k) \\ \omega(t,k+1) \end{bmatrix}^T \Gamma \begin{bmatrix} \varphi(t,k) \\ \omega(t,k+1) \end{bmatrix}\right] \tag{13}$$

Using the Schur complement to Eq. (21) and considering $\begin{bmatrix} Q_1+K_1^TUK_1 & K_1^TUK_2 \\ K_2^TUK_1 & Q_2+K_2^TUK_2 \end{bmatrix} > 0$, we can obtain $\Gamma < 0$, which implies $J_1 < 0$. In other words, if Eq. (21) holds, $\|z(t,k+1)\|_{2D\text{-}2e} \le \gamma\|\omega(t,k+1)\|_{2D\text{-}2e}$ is guaranteed. ■

Theorems 1 and 2 provide sufficient conditions for the existence of 2-D robust guaranteed cost controller that robustly stabilizes the system (7) under repetitive and non-repetitive perturbation, respectively. Such existences ensure the universality and prerequisite for solving the perfect tracking controller for a batch process, even an ordinary 2-D system with interval time delay. That is to say, by above theorems, the ideal controller design confronting interval time delay is no longer a problem. However, we cannot solve matrix inequalities in Eqs. (12) and (15d) directly by numerical method because they are nonlinear. The following subsection provides an easy way to handle this problem.

Robust guaranteed cost control Via 2-D state feedback and system structure

In this section, we will design a 2-D state feedback for system (7) and the cost function (9), such that the resulting closed-loop system (12) is asymptotically stable, and the cost function of the closed-loop system is lower than a specified upper bound.

Theorem 3 Given scalars $0 \le d_m \le d_M$, the robust guaranteed cost control problem of the 2-D system (7) is solvable if symmetric positive matrices L, $\tilde{Q}$, $\bar{Q}$, $\tilde{R}$, $\bar{W}$, $\tilde{W}$, and $X \in R^{(n+l)\times(n+l)}$, matrices Y_1, $Y_2 \in R^{m\times(n+l)}$, and positive scalars ε, $\alpha < 1$ exist, such that the following LMI holds

$$\begin{bmatrix} \Pi & \Omega_1^T & \Omega_2^T & \Omega_3^T & \Omega_4^T & \Omega_5^T & \Omega_6^T & \Omega_7^T & \Omega_8^T \\ * & -\bar{U} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -L+\varepsilon\bar{E}\bar{E}^T & \varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \varepsilon\bar{E}\bar{E}^T & -d_1X+\varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\bar{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -d_2\bar{Q} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -\bar{Q}_1 & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & -\bar{Q}_2 & \mathbf{0} \\ * & * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{23}$$

where

$$\Pi = \begin{bmatrix} -\alpha L-\tilde{R} & \mathbf{0} & \mathbf{0} & L \\ \mathbf{0} & -(1-\alpha)L & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\tilde{Q} & \mathbf{0} \\ L & \mathbf{0} & \mathbf{0} & -\tilde{W}-X \end{bmatrix},$$

$\Omega_1=[Y_1 \quad Y_2 \quad \mathbf{0} \quad \mathbf{0}]$,
$\Omega_2=[\bar{A}_1L+\bar{B}Y_1 \quad \bar{A}_2L+\bar{B}Y_2 \quad \bar{A}_dL \quad \mathbf{0}]$,
$\Omega_3=[\bar{A}_1L+\bar{B}Y_1-L \quad \bar{A}_2L+\bar{B}Y_2 \quad \bar{A}_dL \quad \mathbf{0}]$,
$\Omega_4=\Omega_5=\Omega_6=[L \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}]$,
$\Omega_7=[0 \quad L \quad \mathbf{0} \quad \mathbf{0}]$, $\Omega_8=[\bar{F}L+F_bY_1 \quad F_bY_2 \quad \bar{F}_dL \quad \mathbf{0}]$, and $d_1=d_M^{-2}$, $d_2=(d_M-d_m+1)^{-1}$.

In this case, for any delay $d(t)$ satisfying $0 \le d_m \le d(t) \le d_M$, the guaranteed cost law can be chosen as

$$\begin{aligned} r(t,k+1) &= K_1x_1(t,k+1)+K_2x_1(t+1,k) \\ &= Y_1L^{-1}x_1(t,k+1)+Y_2L^{-1}x_1(t+1,k) \end{aligned} \tag{24a}$$

and the corresponding cost function of the resulting closed-loop 2-D system $\Sigma_{\text{2D-P-delay-C}}$ (12) satisfies

$$\begin{aligned} J \le \sum_{k=0}^{N_2} & [x_1^T(0,k+1)\alpha L^{-1}x_1(0,k+1) \\ &+ \sum_{r=-d(0)}^{-1} x_1^T(r,k+1)\bar{Q}^{-1}x_1(r,k+1) \\ &+ \sum_{r=-d_M}^{-1} x_1^T(r,k+1)\bar{W}^{-1}x_1(r,k+1) \\ &+ \sum_{s=-d_M}^{-d_m}\sum_{r=s}^{-1} x_1^T(r,k+1)\bar{Q}^{-1}x_1(r,k+1) \\ &+ d_M \sum_{s=-d_M}^{-1}\sum_{r=s}^{-1}\eta^T(r,k+\tau)X^{-1}\eta(r,k+\tau)] \\ &+ \sum_{t=0}^{N_1} x_1^T(t+1,0)(1-\alpha)L^{-1}x_1(t+1,0) \end{aligned} \tag{24b}$$

Proof: Premultiply and postmultiply inequality (13b) by $\text{diag}[L,L,L,R^{-1},I,L,R^{-1}]$; define $L=P^{-1}$, $X=R^{-1}$, $W^{-1}=\bar{W}$, $Q^{-1}=\bar{Q}$, $Q_1{}^{-1}=\bar{Q}_1$, $Q_2{}^{-1}=\bar{Q}_2$, $U^{-1}=\bar{U}$, and $Y=[Y_1 \quad Y_2]=[K_1L \quad K_2L]$; and let $\tilde{W}$, $\tilde{Q}$, and $\tilde{R}$ replace $X\bar{W}^{-1}X$, $L\bar{Q}^{-1}L$, and $LX^{-1}L$, respectively. At the same time, applying the Schur complement, the following matrix inequality can be obtained

$$\begin{bmatrix} \Pi & \Omega_1^T & \Omega_{2k}^T & \Omega_{3k}^T & \Omega_4^T & \Omega_5^T & \Omega_6^T & \Omega_7^T \\ * & -\bar{U} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -L & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -d_1X & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\bar{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -d_2\bar{Q} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -\bar{Q}_1 & \mathbf{0} \\ * & * & * & * & * & * & * & -\bar{Q}_2 \end{bmatrix} < 0 \tag{25a}$$

where $\Omega_{2k}=[\tilde{A}_1L+\tilde{B}Y_1 \quad \tilde{A}_2L+\tilde{B}Y_2 \quad \tilde{A}_dL \quad \mathbf{0}]$ and $\Omega_{3k}=[\tilde{A}_1L+\tilde{B}Y_1-L \quad \tilde{A}_2L+\tilde{B}Y_2 \quad \tilde{A}_dL \quad \mathbf{0}]$.

The matrix inequality (25a) contains time-varying matrices $\tilde{A}_1$, $\tilde{A}_d$, and $\tilde{B}$. Similarly, using the Schur complement and Lemma 1,[11] the matrix inequality (25a) holds for any $\Delta(t,k)$satisfying $\Delta^T(t,k)\Delta(t,k) \le I$ if and only if, a scalar $\varepsilon > 0$ exists such that

$$\Upsilon+\varepsilon\Lambda\Lambda^T+\varepsilon^{-1}\Xi^T\Xi < 0 \tag{25b}$$

where

$$\Upsilon = \begin{bmatrix} \Pi & \Omega_1^T & \Omega_2^T & \Omega_3^T & \Omega_4^T & \Omega_5^T & \Omega_6^T & \Omega_7^T \\ * & -\bar{U} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -L & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -d_1X & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\bar{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -d_2\bar{Q} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -\bar{Q}_1 & \mathbf{0} \\ * & * & * & * & * & * & * & -\bar{Q}_2 \end{bmatrix}$$

$$\Lambda=[\mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \bar{E}^T \quad \bar{E}^T \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}]^T$$

$$\Xi=[\bar{F}L+F_bY_1 \quad F_bY_2 \quad \bar{F}_dL \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}]$$

Applying the Schur complement, the above inequality (25b) is equal to the matrix inequality (23). ■

Theorem 4 For a prescribed constant $\gamma > 0$ and given scalars $0 \leq d_m \leq d_M$, the robust H_∞ guaranteed cost control problem of the 2-D system (7) is solvable if symmetric positive matrices L, $\tilde{Q}$, $\bar{Q}$, $\tilde{R}$, $\bar{W}$, $\tilde{W}$, $X \in R^{(n+l)\times(n+l)}$, matrices Y_1, $Y_2 \in R^{m\times(n+l)}$, and positive scalars ε, $\alpha < 1$ exist, such that the following LMI holds:

$$\begin{bmatrix} \bar{\Pi} & \bar{\Omega}_1^T & \bar{\Omega}_2^T & \bar{\Omega}_3^T & \bar{\Omega}_4^T & \bar{\Omega}_5^T & \bar{\Omega}_6^T & \bar{\Omega}_7^T & \bar{\Omega}_8^T & \breve{G}^T \\ * & -\bar{U} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -L+\varepsilon\bar{E}\bar{E}^T & \varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \varepsilon\bar{E}\bar{E}^T & -d_1X+\varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\bar{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -d_2\bar{Q} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -\bar{Q}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & -\bar{Q}_2 & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & * & -\varepsilon I & \mathbf{0} \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (26)$$

where

$\bar{\Pi}=\begin{bmatrix} \Pi & \mathbf{0} \\ \mathbf{0} & -\gamma^2 I \end{bmatrix}$, $\bar{\Omega}_1=[\Omega_1 \quad \mathbf{0}]$, $\bar{\Omega}_2==[\Omega_2 \quad \tilde{H}]$, $\bar{\Omega}_3=[\Omega_3 \quad \tilde{H}]$, $\bar{\Omega}_4=\bar{\Omega}_5=\bar{\Omega}_6=[\Omega_4 \quad \mathbf{0}]$, $\bar{\Omega}_7=[\Omega_7 \quad \mathbf{0}]$, $\bar{\Omega}_8=[\Omega_8 \quad \mathbf{0}]$, and $\breve{G}=[\tilde{G}L \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}]$.

On the other hand, the robust H_∞ guaranteed cost control law can still be chosen as (24a), and the corresponding cost function of the resulting closed-loop 2-D system $\Sigma_{\text{2D-P-delay-C}}$ (12) still satisfies (24b).

Proof: Premultiply and postmultiply inequality (21) by $\text{diag}[L, L, L, R^{-1}, I, I, L, R^{-1}, I]$, and from the proof of Theorem 3, Theorem 4 can be obtained immediately. ■

Theorems 3 and 4 are sufficient conditions for solving the 2-D robust guaranteed cost controller that robustly stabilizes the system (7). Combined with the algorithm in subsection "Performance optimization," we successfully achieve the design of the controller. In addition, the closed-loop system (12) is depicted by Figure 1, where the dotted arrows denote the information flow of the previous cycle from the storages and the solid arrows represent the flow of real-time feedback information. This block diagram can be explained from two different angles: one is a 2-D time-delay system; the other is a batch process. From the perspective of a 2-D system, the block diagram is a 2-D state feedback control system that comprises a 2-D plant $\Sigma_{\text{2D-P-delay}}$ and a 2-D state feedback controller Σ_{C}. From the perspective of a batch process, the system consists of a plant $\Sigma_{\text{P-delay}}$ and a 2-D ILC law $\Sigma_{\text{2D-C-delay}}$. From Eqs. (4) and (10), the 2-D ILC law $\Sigma_{\text{2D-C-delay}}$ can be decomposed as follows

$$\Sigma_{\text{2D-C-delay}} : u(t,k+1)=u_l(t,k)+u_r(t,k+1) \quad (27)$$

where $u_l(t,k+1)=u(t,k)+K_2x_1(t+1,k)$ is an ILC law for the performance improvement along the cycle direction, and $u_r(t,k+1)=K_1x_1(t,k+1)$ is a real-time state feedback control law for ensuring control performance over time.

The results of the above theorems are all considered based on the interval time-delay situation. In the referred study, the time-delay $d(t)$ was often assumed to be zero or fixed. Therefore, without losing generality, we will also consider

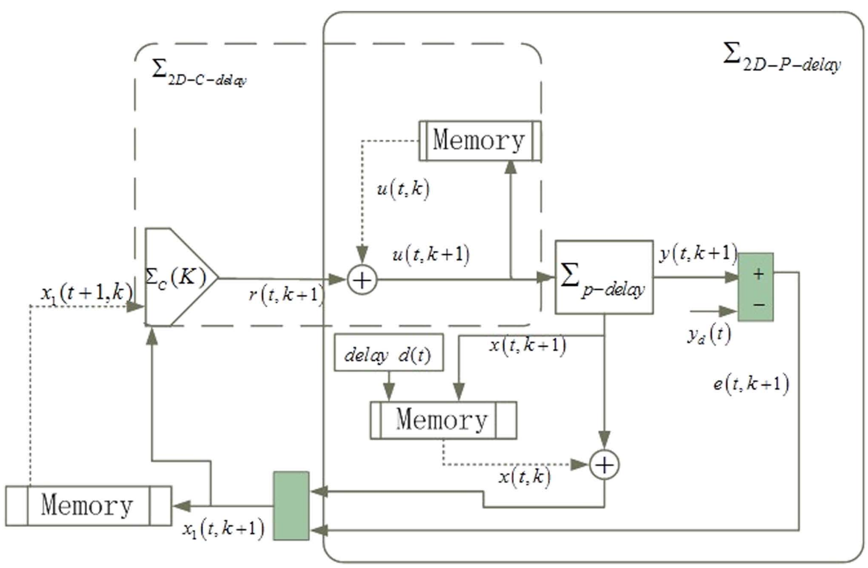

Figure 1. Schematic diagram of the structure of a closed-loop system.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

the constant delay case in this paper. First, the system (7) is deduced to the following form

$$\sum\nolimits'_{\text{2D-P-delay}}: \begin{cases} x_1(t+1,k+1)=\tilde{A}_1x_1(t,k+1)+\tilde{A}_2x_1(t+1,k) \\ +\tilde{A}_dx_1(t-d,k+1)+\tilde{B}r(t,k+1)+\tilde{H}\omega(t,k+1) \\ z(t,k+1)\triangleq e(t,k+1)=\tilde{G}x_1(t,k+1) \end{cases} \tag{28a}$$

Accordingly, the resulting closed-loop 2-D system $\sum'_{\text{2D-P-delay-C}}$ is rewritten as

$$\sum\nolimits'_{\text{2D-P-delay-C}}: \begin{cases} x_1(t+1,k+1)=\tilde{A}_{1k}x_1(t,k+1)+\tilde{A}_{2k}x_1(t+1,k) \\ \qquad +\tilde{A}_dx_1(t-d,k+1)+\tilde{H}\omega(t,k+1) \\ \qquad z(t,k+1)\triangleq e(t,k+1)=\tilde{G}x_1(t,k+1) \end{cases} \tag{28b}$$

The above theorems will then be transformed into more general constant delay results. The following corollaries would be obtained.

Corollary 1. For a given integer $\bar{d}>0$, the 2-D control law $r(t,k+1)=K_1x_1(t,k+1)+K_2x_1(t+1,k)$ is a guaranteed cost controller if symmetric positive matrices P, W, and $R\in R^{(n+l)\times(n+l)}$ and a positive scale $\alpha<1$ exist, such that the following matrix inequality holds, as shown in (29b), and the cost function (9) of the resulting closed-loop 2-D system $\sum'_{\text{2D-P-delay-C}}$ (28b) satisfies

$$\begin{aligned} J\le\sum_{k=0}^{N_2}&[x_1^T(0,k+1)\alpha Px_1(0,k+1)+\sum_{r=-\bar{d}}^{-1}x_1^T(r,k+1)Wx_1(r,k+1) \\ &+\bar{d}\sum_{s=-\bar{d}}^{-1}\sum_{r=s}^{-1}\eta^T(r,k+1)R\eta(r,k+1)] \\ &+\sum_{t=0}^{N_1}x_1^T(t+1,0)(1-\alpha)Px_1(t+1,0) \end{aligned} \tag{29a}$$

$$\begin{bmatrix} \tilde{\Psi}_1 & \tilde{\Psi}_2 \\ \tilde{\Psi}_2^T & \tilde{\Psi}_3 \end{bmatrix}<0 \tag{29b}$$

where $\tilde{\Psi}_1=\begin{bmatrix} -\alpha P+W+Q_1-R & \mathbf{0} & R \\ \mathbf{0} & -(1-\alpha)P+Q_2 & \mathbf{0} \\ R & \mathbf{0} & -W-R \end{bmatrix}$,

$\tilde{\Psi}_2=\begin{bmatrix} K_1^T & \tilde{A}_{1k}^TP & (\tilde{A}_{1k}-I)^TR \\ K_2^T & \tilde{A}_{2k}^TP & \tilde{A}_{2k}^TR \\ \mathbf{0} & \tilde{A}_d^TP & \tilde{A}_d^TR \end{bmatrix}$, and

$\tilde{\Psi}_3=-\text{diag}[U^{-1} \quad P \quad \bar{d}^{-2}R]$.

Proof: Choose the following Lyapunov function candidate

$$V'(t+\theta,k+\tau)=V'_h(t+\theta,k+\tau)+V'_v(t+\theta,k+\tau) \tag{30}$$

where
$V'_h(t+\theta,k+\tau)=\Sigma_{l=1}^3V'_l(t+\theta,k+\tau)$,
$V'_1(t+\theta,k+\tau)=x_1^T(t+\theta,k+\tau)\alpha Px_1(t+\theta,k+\tau)$,
$V'_2(t+\theta,k+\tau)=\Sigma_{r=t+\theta-d}^{t+\theta-1}x_1^T(r,k+\tau)Wx_1(r,k+\tau)$,
$V'_3(t+\theta,k+\tau)=d\Sigma_{s=-d}^{-1}\Sigma_{r=t+\theta+s}^{t+\theta-1}\eta^T(r,k+\tau)R\eta(r,k+\tau)$,
$V_v(t+\theta,k+\tau)=x_1^T(t+\theta,k+\tau)(1-\alpha)Px_1(t+\theta,k+\tau)$, and P, W, and R are the positive definite matrices to be determined. The corollary can then be obtained using lines similar to those in the proof of Theorem 1. ■

Corollary 2. For a prescribed constant $\gamma>0$ and a given integer $\bar{d}>0$, the 2-D control law $r(t,k+1)=K_1x_1(t,k+1)+K_2x_1(t+1,k)$ is a robust H_∞ guaranteed cost control law for system (28a), and the cost function satisfies (29a) if symmetric positive matrices P, W, and $R\in R^{(n+l)\times(n+l)}$; and a positive scale $\alpha<1$ exist, such that the following matrix inequality holds

$$\begin{bmatrix} \breve{\Psi}_1 & \breve{\Psi}_2 & \breve{G}^T \\ \breve{\Psi}_2^T & \tilde{\Psi}_3 & \mathbf{0} \\ \breve{G} & \mathbf{0} & -I \end{bmatrix}<0 \tag{31}$$

where $\breve{\Psi}_1=\begin{bmatrix} \tilde{\Psi}_1 & \mathbf{0} \\ \mathbf{0} & -\gamma^2I \end{bmatrix}$, $\breve{\Psi}_2=\begin{bmatrix} \tilde{\Psi}_2 \\ \hat{H} \end{bmatrix}$, and $\breve{G}=[\tilde{G} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}]$.

Corollary 3. Given an integer $\bar{d}$, the robust guaranteed cost control problem of the 2-D system (28a) is solvable if symmetric positive matrices L, $\tilde{R}$, $\bar{W}$, $\tilde{W}$ and $X\in R^{(n+l)\times(n+l)}$, matrices Y_1 and $Y_2\in R^{m\times(n+l)}$, and positive scalars ε and $\alpha<1$ exist such that the following LMI holds

$$\begin{bmatrix} \tilde{\Pi} & \tilde{\Omega}_1^T & \tilde{\Omega}_2^T & \tilde{\Omega}_3^T & \tilde{\Omega}_4^T & \tilde{\Omega}_6^T & \tilde{\Omega}_7^T & \tilde{\Omega}_8^T \\ * & -\bar{U} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -L+\varepsilon\bar{E}\bar{E}^T & \varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \varepsilon\bar{E}\bar{E}^T & -\bar{d}^{-2}X+\varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\bar{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -\bar{Q}_1 & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -\bar{Q}_2 & \mathbf{0} \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix}<0 \tag{32}$$

where
$\tilde{\Pi}=\begin{bmatrix} -\alpha L-\tilde{R} & \mathbf{0} & L \\ \mathbf{0} & -(1-\alpha)L & \mathbf{0} \\ L & \mathbf{0} & -\tilde{W}-X \end{bmatrix}$,
$\tilde{\Omega}_1=[Y_1 \; Y_2 \; \mathbf{0}]$, $\tilde{\Omega}_2=[\bar{A}_1L+\bar{B}Y_1 \; \bar{A}_2L+\bar{B}Y_2 \; \bar{A}_dL]$,
$\tilde{\Omega}_3=[\bar{A}_1L+\bar{B}Y_1-L \; \bar{A}_2L+\bar{B}Y_2 \; \bar{A}_dL]$,
$\tilde{\Omega}_4=\tilde{\Omega}_6=[L \; \mathbf{0} \; \mathbf{0}]$,
$\tilde{\Omega}_7=[\mathbf{0} \; L \; \mathbf{0}]$, and $\tilde{\Omega}_8=[\bar{F}L+F_bY_1 \; F_bY_2 \; \bar{F}_dL]$.

In this case, for any delay d satisfying $0\le d\le\bar{d}$, the guaranteed cost law is chosen as (23), but the corresponding cost function of the resulting closed-loop 2-D system (28b) satisfies

$$\begin{aligned} J\le\sum_{k=0}^{N_2}&[x_1^T(0,k+1)\alpha L^{-1}x_1(0,k+1) \\ &+\sum_{r=-\bar{d}}^{-1}x_1^T(r,k+1)\bar{W}^{-1}x_1(r,k+1) \\ &+\bar{d}\sum_{s=-\bar{d}}^{-1}\sum_{r=s}^{-1}\eta^T(r,k+\tau)X^{-1}\eta(r,k+\tau)] \\ &+\sum_{t=0}^{N_1}x_1^T(t+1,0)(1-\alpha)L^{-1}x_1(t+1,0) \end{aligned} \tag{33}$$

Corollary 4. For a prescribed constant $\gamma>0$ and a given integer $\bar{d}$, the robust H_∞ guaranteed cost control problem of the 2-D system (28a) is solvable, and the corresponding cost function of the resulting closed-loop 2-D system (28b) satisfies (33) if symmetric positive matrices L, $\tilde{R}$, $\bar{W}$, $\tilde{W}$, and $X\in R^{(n+l)\times(n+l)}$, matrices Y_1 and $Y_2\in R^{m\times(n+l)}$, and positive scalars ε and $\alpha<1$ exist such that the following LMI holds

$$\begin{bmatrix} \widehat{\Pi} & \widehat{\Omega}_1^T & \widehat{\Omega}_2^T & \widehat{\Omega}_3^T & \widehat{\Omega}_4^T & \widehat{\Omega}_6^T & \widehat{\Omega}_7^T & \widehat{\Omega}_8^T & \vec{G}^T \\ * & -\bar{U} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -L+\varepsilon\bar{E}\bar{E}^T & \varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \varepsilon\bar{E}\bar{E}^T & -\bar{d}^{-2}X+\varepsilon\bar{E}\bar{E}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\bar{W} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -\bar{Q}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & -\bar{Q}_2 & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & -\varepsilon I & \mathbf{0} \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{34}$$

where $\widehat{\Pi} = \begin{bmatrix} \tilde{\Pi} & \mathbf{0} \\ \mathbf{0} & -\gamma^2 I \end{bmatrix}$, $\widehat{\Omega}_1 = [\tilde{\Omega}_1 \quad \mathbf{0}]$, $\widehat{\Omega}_2 == [\tilde{\Omega}_2 \quad \tilde{H}]$, $\widehat{\Omega}_3 = [\tilde{\Omega}_3 \quad \tilde{H}]$, $\widehat{\Omega}_4 = \widehat{\Omega}_6 = [\tilde{\Omega}_4 \quad \mathbf{0}]$, $\widehat{\Omega}_7 = [\tilde{\Omega}_7 \quad \mathbf{0}]$, $\widehat{\Omega}_8 = [\tilde{\Omega}_8 \quad \mathbf{0}]$, and $\vec{G} = [\tilde{G}L \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}]$.

The proofs of the corollaries are similar to those of the theorems, making them omitted here.

The above corollaries develop sufficient conditions for existence and solvability of 2-D robust guaranteed cost controller that robustly stabilizes the system (28a) with fixed delay under repetitive and non repetitive perturbation cases, respectively. Different from the existing results, here we avoid introducing some redundant free weighting matrices that apart from the Lyapunov–Krasovskii functional matrices. Thereby, our results naturally have fewer constraints and a smaller amount of computation. To obtain the guaranteed cost controller, the following optimization algorithm must be associated with the above results.

Performance optimization

The upper bound of cost function in Theorem 3 has been noted to be dependent on the initial condition (8) of system (7). To remove this dependence on the initial condition, we have assumed that the cost function has a finite set of initial conditions, that is, two positive integers t and k exist, such that

$$x_1(t,0)=0,\ t \geq r_1; \quad x_1(0,k)=0,\ k \geq r_2 \tag{35}$$

For any $0 \leq t < r_1$ and $0 \leq k < r_2$, we suppose that the initial state is arbitrary but belongs to the set

$$\begin{aligned} \mathbb{S}(d_M,0) = \{x_1(t,0) \in R^n : x_1(t,0) = \Theta v_t, v_t^T v_t \leq I, 0 \leq t < r_1\} \\ \cup \{x_1(0,k) \in R^n : x_1(0,k) = \Theta v_k, v_i^T v_i \leq I, 0 \leq k < r_2\} \end{aligned} \tag{36}$$

where Θ is a given matrix. The cost bound (9) then yields

$$\begin{aligned} J \leq r_2 \Bigg[\alpha\beta + d_M\gamma_1 + d_M\gamma_2 + \frac{(d_M+d_m)(d_M-d_m)}{2}\gamma_1 \\ + \frac{d_M^2(d_M+1)}{2}\gamma_3 \Bigg] + r_1(1-\alpha)\beta \end{aligned} \tag{37}$$

where

$$\begin{bmatrix} -\beta I & \Theta^T \\ \Theta & -L \end{bmatrix} < 0, \begin{bmatrix} -\gamma_1 I & \Theta^T \\ \Theta & -\bar{Q} \end{bmatrix} < 0, \begin{bmatrix} -\gamma_2 I & \Theta^T \\ \Theta & -\bar{W} \end{bmatrix} < 0, \begin{bmatrix} -\gamma_3 I & \Theta^T \\ \Theta & -X \end{bmatrix} < 0 \tag{38}$$

To obtain the controller $r(t,k+1) = Y_1 L^{-1} x_1(t,k+1) + Y_2 L^{-1} x_1(t+1,k)$ and achieve the least guaranteed cost value J^*, we have to solve the following optimization problem

$$\begin{aligned} \min r_2 \Bigg[\alpha\beta + d_M\gamma_1 + d_M\gamma_2 + \frac{(d_M+d_m)(d_M-d_m)}{2}\gamma_1 \\ + \frac{d_M^2(d_M+1)}{2}\gamma_3 \Bigg] + r_1(1-\alpha)\beta \end{aligned} \tag{39}$$

$$\text{s.t. } 0 < \alpha < 1, (23), (38) \tag{40}$$

On the other hand, in the matrix inequality (23), the conditions are no longer LMI conditions because of the terms $L\bar{Q}^{-1}L$, $LX^{-1}L$, and $L\bar{W}^{-1}L$. To solve the problem, we introduce new variables, $\tilde{Q}$, $\tilde{R}$, and $\tilde{W}$, in Theorem 3 and transform them into some inequalities that are similar to Eq. (13). Following a similar step as in Lee et al.,[9] the original nonconvex feasibility problem characterized by the matrix inequality (23) is then converted to the nonlinear minimization problem subject to LMI. By utilizing the linearization method,[29] when $0 < \alpha < 1$ is given, the above optimization problem is a convex optimization problem (40) that can be solved by the solver minicx in the LMI toolbox. Furthermore, we can determine the optimal scalar α^*, such that the guaranteed cost bound (24b) is minimized.

Remark 4. To get the upper bound of the optimal performance, we first give the assumptions on the initial state of the system, which satisfy conditions (35) and (36). These conditions are essential for solving system performance, such as, considering guaranteed cost control to the thermal processes in chemical reactors, heat exchangers, and pipe furnaces. To obtain the minimum performance value J^*, the following efforts need to do: (a) choose adequate the value of α. If α is not given and to be regarded as decision variables, (23) is a nonlinear matrix inequality, sufficient condition (23) is not an LMI which cannot be solved by using LMI tools. (b) How to adjust the value of α is essential. The size of value α will directly affect the matrix inequality's solution. First given larger α, solve the constraints (40); if there is a feasible solution, then given smaller α, go on; otherwise stop. The final goal is to obtain the minimum value J^*.

Remark 5. As can be seen from a given objective function, the size of value J^* is determined by the upper and lower bounds of delay. This is different from the results in the pertinent literature, in which the delay is usually fixed. In fact, interval delay may degenerate into constant delay. Thus, the results in this article are universal, which includes the constant delays situations as a special case. In addition, the corresponding results have been expressed by the following corollaries. It is easy to see these results are applicable to general 2-D systems. This will once again prove the results universal.

Although the upper bound of the cost function in Theorem 4 is similar to that in Theorem 3, the constraint condition is different from that in Theorem 3. The optimization problem is

$$\min r_2\left[\alpha\beta+d_M\gamma_1+d_M\gamma_2+\frac{(d_M+d_m)(d_M-d_m)}{2}\gamma_1 + \frac{d_M^2(d_M+1)}{2}\gamma_3\right]+r_1(1-\alpha)\beta \tag{41}$$

$$\text{s.t. } 0<\alpha<1, (26), (38) \tag{42}$$

For the LMI described by Eq. (26), $\gamma > 0$ can be further regarded as an optimization variable. The design objective is such that γ is as small as possible. At the same time, we will determine the optimal scalar α^*, such that the guaranteed cost bound (24b) is minimized. The ideal α^* can be obtained using the following method: given a larger γ, solve the inequalities (42); if a feasible solution exists, then proceed given a smaller γ; otherwise, stop.

Remark 6. The above optimization problem (39) with constraints (40) is designed for repetitive disturbance**.** When parameter perturbations are nonrepetitive, as described in Remark 2, the robust H_∞ control problem is considered to analyze the sensitivity of the controlled output to the disturbance. The optimization problem (41) with constraints (42) is proposed in this case. From the constraints, we can see that the sizes of values of γ and α together affect the LMIs (42) whether they are solvable or not. At the same time, performance index γ need to be as small as possible. The final goal is still to obtain the minimum value J^*.

Regarding the constant delay case, using lines similar to those in the proof of Theorem 2, the corollaries can be easily obtained. On the other hand, for the initial state belonging to the set (36), the cost bound (33) yields

$$J \le r_2\left[\alpha\beta+\bar{d}\gamma_2+\frac{\bar{d}^2(\bar{d}+1)}{2}\gamma_3\right]+r_1(1-\alpha)\beta \tag{43}$$

where

$$\begin{bmatrix} -\beta I & \Theta^T \\ \Theta & -L \end{bmatrix}<0, \begin{bmatrix} -\gamma_2 I & \Theta^T \\ \Theta & -\bar{W} \end{bmatrix}<0, \begin{bmatrix} -\gamma_3 I & \Theta^T \\ \Theta & -X \end{bmatrix}<0 \tag{44}$$

The optimization problem returns to

$$\min r_2\left[\alpha\beta++\bar{d}\gamma_2+\frac{\bar{d}^2(\bar{d}+1)}{2}\gamma_3\right]+r_1(1-\alpha)\beta \tag{45}$$

$$\text{s.t. } 0<\alpha<1, (32), (44) \tag{46}$$

The solution process is similar to the above algorithm; here, it is omitted. Corollary 4 is likewise omitted.

Illustration

This section applies the main results on guaranteed cost control to a class of batch processes with the form Eq. (27), which widely exists in numerous chemical processes, such as the barrel temperature in the batch process. Here, the time-delay is a state delay varying in an interval, which implies that the system status information is related not only to the current time information but also to the previous moment. Variations in working conditions may make the process appear as a batch process with uncertain parameter perturbations. This condition limits the applicability of the conventional ILC. Pure feedback control, such as proportional integrative derivative control and adaptive control, cannot improve control performance from cycle to cycle. Designing a robust 2-D controller that can ensure the improvement of both the performance over time and the tracking performance from cycle to cycle is necessary. Consider the system with the following form

$$\sum\nolimits_{\text{P-delay}}: \begin{cases} x(t+1,k+1)=(A+E\Delta(t,k)F)x(t,k+1) \\ \qquad +(A_d+E\Delta(t,k)F_d)x(t-d(t),k+1) \\ \qquad +(B+E\Delta(t,k)F_b)u(t,k+1) \\ y(t,k)=Cx(t,k) \end{cases} \tag{47}$$

where $A=\begin{bmatrix} 1.607 & 1 \\ -0.6916 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 1.239 \\ -0.3282 \end{bmatrix}$, $C=[1 \quad 0]$, $E\Delta(t)F$, $E\Delta(t)F_d$, and $E\Delta(t)F_b$ are parameter perturbations with $E=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $F=\begin{bmatrix} 0.08 & 0 \\ 0.08 & 0 \end{bmatrix}$, $F_d=\begin{bmatrix} 0.02 & 0 \\ 0.01 & 0 \end{bmatrix}$, $F_b=\begin{bmatrix} 0.1 \\ 0.14 \end{bmatrix}$, $\Delta(t)=\begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}$, $|\delta_1|\le 1$, $|\delta_2|\le 1$, and $A_d=\begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix}$. $x(t,\ k+1)$ indicates the temperature at t (time) and $(k+1)$th (cycle), and $u(t,\ k+1)$ is given force function. Note that the process state delay, $d(t)$, was often assumed to be zero or fixed for ILC design in the referred articles.[2–4,27,28] Only in Refs. [16,17], and [20], the interval time-varying delay is used for controller design of the batch process with uncertainties. Yet those are not considering the designing of a guaranteed adequate level of performance, which is more important because such issue is always desirable especially when controlling a system depending on uncertain parameters. Here, we design a controller described by (10), which not only guarantees stability of the batch process but also makes the system preserving the best performance level. Assume that the initial state satisfies condition (36) for $r_1 = r_2 = 10$ and belongs to the set $\mathbb{S}(d_M, 0)$, where

$$\Theta=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{48}$$

Choose the weighting matrices

$$Q_1=Q_2=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U=1 \tag{49}$$

According to the following two different cases, we can then obtain the control objective.

Case 1. Repetitive uncertainty

In this simulation case, parameter perturbations, $\{\delta_i : |\delta_i| < 1\}_{i=1,2}$, are repetitive along the batch direction and are assumed to vary randomly within [0, 1] along the time direction. For $d_m = 1$ and $d_M = 3$, assume that the state delay varies randomly in a range as estimated above. By solving Eq. (40), the guaranteed cost J^* via different α is shown in Figure 2. When $\alpha = 0.4001$, the least upper bound of the corresponding closed-loop cost function is $J^* = 143.2223$ for the resulting closed-loop system. Meanwhile, the optimal guaranteed cost controller

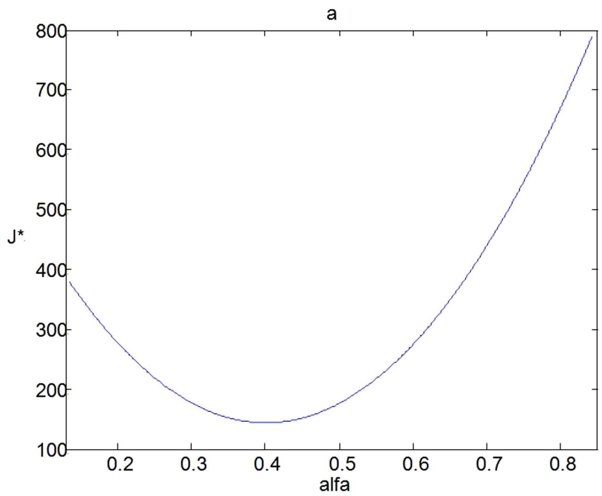

Figure 2. Guaranteed cost J^* via different α.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

$$r^*(t,k+1)=[-1.3267\quad -0.8403\quad -0.0000]x_1(t,k+1) + [-0.0000\quad -0.0000\quad -0.4421]x_1(t+1,k) \tag{50}$$

can stabilize system (47). The tracking results are shown in Figure 4. The output of the system tracks rapidly to a given output over time, although steady-state tracking errors exist in the cycles.

Case 2. Nonrepetitive uncertainty

In this case, parameter perturbations, $\{\delta_i : |\delta_i| < 1\}_{i=1,2}$, are taken as random variables within [0, 1], but are made nonrepeatable along the batch direction. Other elements remain fixed, as in Case 1. A γ-suboptimal 2-D H_∞ guaranteed cost controller for system (47) is designed along the following line: First, select a larger γ, and solve Eq. (42); if α can be obtained, then a smaller γ is chosen. The process is continued until we find a suitable γ, such that Eq. (42) is solvable for the smallest α. The guaranteed costs J^* via different α are shown in Figure 3. When $\gamma = 9.1$ and $\alpha =$

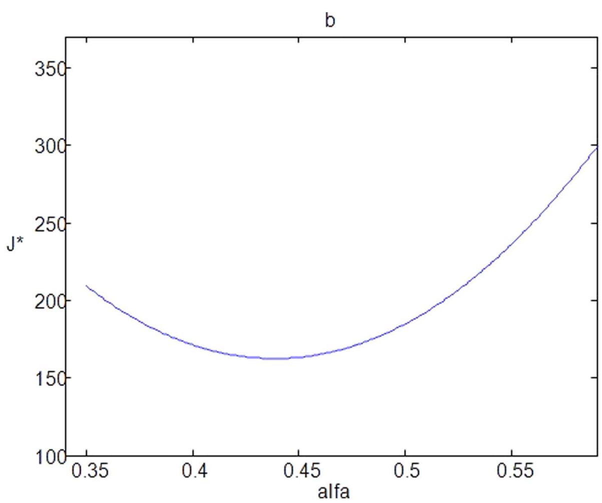

Figure 3. For an available γ = 9.1, the guaranteed cost J^* via different α.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

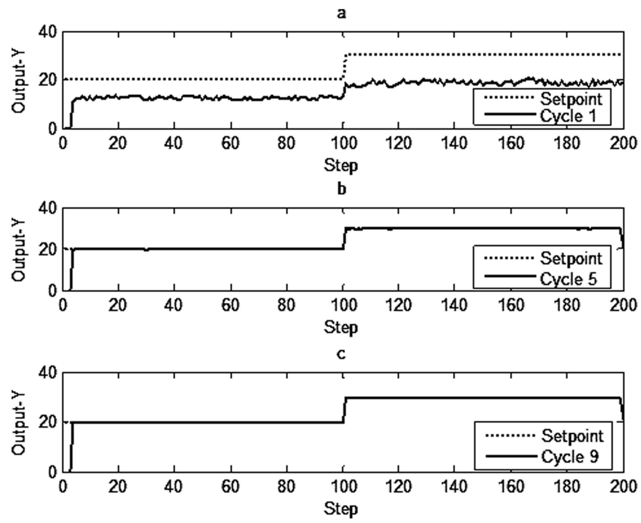

Figure 4. Output responses of Case 1.

0.4011, the least guaranteed cost is $J^* = 171.0892$. The corresponding optimal guaranteed cost controller is

$$r^*(t,k+1)=[-1.3214\quad -0.8460\quad -0.0000]x_1(t,k+1) + [-0.0000-0.0000-0.4505]x_1(t+1,k) \tag{51}$$

Based on these results, when the system is affected by the interference, the system performance indeed decreases. Figure 5 also demonstrates the effect, showing the tracking results of representative cycles of 1, 5, and 9 correspondingly. The proposed method holds the good robustness and convergence of the designed control system against some degree of nonrepetitive uncertainty using the above controller although system performance is decreased.

Remark 7. The above theorems and corollaries give sufficient conditions for the existence and solvability of the proposed controller according to different delay cases and different perturbation cases, respectively. By optimizing the design algorithm, two kinds of controller are derived. As we all know, for repeated perturbations, the 2-D control strategy by the combination of feedback and ILC has a good solution to the convergence of the batch process along time and batch direction. Its control effect can be seen from Figure 4. At

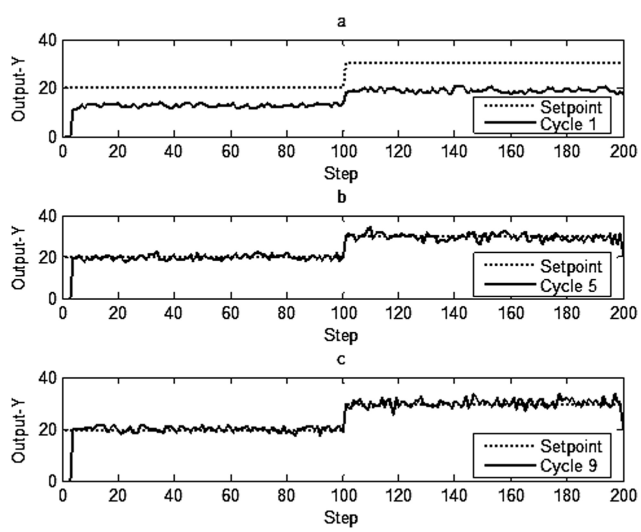

Figure 5. Output responses of Case 2.

the initial stage, the output of the system does not track on a given output. With the control input intervention, the tracking performance is improved from cycle to cycle. It leads to perfect tracking after several cycles. For nonrepetitive perturbations, we just make internal disturbances of the system (1) converted to external disturbances, and thereby take into account H_∞ control problem. In other words, to ensure that the system has good robust performance, in the design of the algorithm, performance index γ should be as small as possible. From Figure 5, we can see that although the designed controller cannot eliminate the influence of cycle-to-cycle parameter perturbations, the robust H_∞ performance can be guaranteed.

Conclusions

In this article, a solution to the optimal guaranteed cost control problem via a robust 2-D controller-based feedback and ILC is presented for a batch process with interval time-varying delay in an LMI framework. Based on an equivalent 2-D time-delay system description of a batch process, a robust 2-D ILC scheme is developed for the robust tracking of a desired trajectory and the preservation of an adequate level of performance. Given the upper and lower bounds of state delay variation, new delay-range-dependent sufficient conditions of the existence of such guaranteed cost controllers are established for assessing the robust stability of the closed-loop system by defining a 2-D Lyapunov function and using the 2-D system theory as basis. By solving the corresponding LMI constraints, the delay-dependent suboptimal robust 2-D guaranteed cost controller is explicitly formulated, along with adjustable robust control performance levels. On the other hand, the results have been extended to the constant delay case. The application to a simulation example with different perturbation cases has evidently demonstrated the effectiveness and merits of the proposed ILC method.

Acknowledgments

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